EXAM COMPUTER VISION, INMCV-08

November 8, 2010, 14:00 hrs

During the exam you may use the lab manual, copies of sheets, **provided they do not contain any notes.**

Put your name on all pages which you hand in, and number them. Write the total number of pages you hand in on the first page. Write clearly and not with pencil or red pen. ***Always motivate your answers.*** *Good luck!*

Problem 1. **(2.0 pt)** Consider an image E with grey level distribution given by

$$E(u_0, v_0) = A \sin(\omega u_0) \tag{1}$$

at $t = 0$ (u_0 and v_0 denote position at $t = 0$), with ω the spatial frequency of the pattern, and A the amplitude. The motion field in the image plane is given by

$$\dot{u} = 1, \quad \dot{v} = 2. \tag{2}$$

a. (0.5 pt) Give an expression for $E(u, v, t)$ (**Hint:** first obtain expressions for $u(t)$ and $v(t)$).

b. (0.5 pt) Compute the observed temporal changes in irradiance $\frac{\partial E}{\partial t}$ as a function of u and v (**Hint:** if you did not solve part a., use the Horn-Schunck equation).

c. (1.0 pt) Given the temporal changes (optic flow) obtained in part b., can the motion field be recovered completely using the Horn-Schunck equation? If not, would addition of smoothness constraints solve this problem? Explain your answer.

Problem 2. **(2.5 pt)** Consider the following inference problem. Given a perspective projection of a cube with three sets of four parallel ribs each, with unknown orientations $\vec{w}^{(X)}$, $\vec{w}^{(Y)}$ and $\vec{w}^{(Z)}$, and three corresponding vanishing points X, Y, Z in the projection plane, see Fig. 1. Two of these points are known $(u_\infty^{(Y)}, v_\infty^{(Y)}) = (1, 2)$, $(u_\infty^{(Z)}, v_\infty^{(Z)}) = (0, -2)$. The camera constant f is unknown.

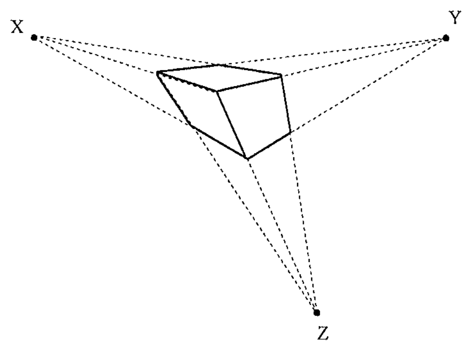

Figure 1: *Perspective projection of a cube with three vanishing points.*

Compute the three orientation vectors $\vec{w}^{(X)}$, $\vec{w}^{(Y)}$, $\vec{w}^{(Z)}$.
Hint: First compute the camera constant f.

Problem 3. (2.5pt) A binary area opening Γ_λ is a connected attribute filter, which removes all connected components in an image with an area smaller than some threshold parameter $\lambda \in \mathbb{R}$. The grey scale counterpart γ_λ is computed by threshold superposition:

$$(\gamma_\lambda(f))(x) = \vee\{h \leq f(x) | x \in \Gamma_\lambda(X_h(f))\} \tag{3}$$

in which $X_h(f)$ is the threshold set at level h, given by

$$X_h(f) = \{x \in E | f(x) \geq h\}. \tag{4}$$

These grey-level area openings γ_λ remove any regional maxima smaller than a given area λ. We will study the set of area openings $\{\gamma_\lambda\}$ indexed by area threshold λ from $\mathbb{R}^+$.

Consider the six axioms of scale spaces, and the three for granulometries.

a. (0.5 pt) Argue that all γ_λ are anti-extensive, i.e., $(\gamma_\lambda(f))(x) \leq f(x)$

b. (0.5pt) Argue that all γ_λ are rotation invariant

c. (1.0pt) Show that the absorbtion property holds, i.e:

$$\gamma_s(\gamma_t(f)) = \gamma_{\mathsf{Max}(s,t)}(f) \qquad \forall s, t \geq 0.$$

d. 0.5pt Argue that this granulometry is contrast invariant in the sense that

$$\gamma_\lambda(\alpha f) = \alpha\gamma_\lambda(f).$$

with α an arbitrary positive constant.

Problem 4. (2.0 pt) Consider a spherical surface of radius r centered at the origin with equation

$$z = d - \sqrt{r^2 - x^2 - y^2}, \qquad x^2 + y^2 \leq r \tag{5}$$

The surface is Lambertian with constant albedo $\rho_S = 1$, and is illuminated by a light source at a very large distance, from a direction defined by the unit vector (a, b, c), with c negative. The camera is on the negative z-axis. Show that the image intensity under orthographic projection is given by

$$E(x, y) = \frac{ax + by - c\sqrt{r^2 - x^2 - y^2}}{r} \tag{6}$$

If we measure the above light distribution in an image, and know the surface is Lambertian, can we then infer that the surface has the form in (5)? Explain your answer.